

Code: 23BS1101

**I B.Tech - I Semester – Regular / Supplementary Examinations  
DECEMBER 2024**

**LINEAR ALGEBRA & CALCULUS  
(Common for ALL BRANCHES)**

Duration: 3 hours

Max. Marks: 70

- Note: 1. This question paper contains two Parts A and B.  
 2. Part-A contains 10 short answer questions. Each Question carries 2 Marks.  
 3. Part-B contains 5 essay questions with an internal choice from each unit. Each Question carries 10 marks.  
 4. All parts of Question paper must be answered in one place.

BL – Blooms Level

CO – Course Outcome

**PART – A**

		BL	CO
1.a)	Define Rank of a matrix.	L1	CO1
1.b)	Describe consistency conditions for Non - Homogeneous Linear system of equations.	L1	CO1
1.c)	If $A = \begin{bmatrix} -1 & 2 & 3 \\ 0 & 3 & 5 \\ 0 & 0 & -2 \end{bmatrix}$ , then find the eigen values of $A^3$ .	L3	CO2
1.d)	Calculate the sum and product of the eigen values of $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & -1 \\ 0 & -1 & 3 \end{bmatrix}$ .	L3	CO2
1.e)	State Lagrange's mean value theorem.	L1	CO5
1.f)	Write the Maclaurin's series of $f(x) = e^x$	L3	CO3
1.g)	If $u = e^{xyz}$ , find the value of $\frac{\partial^3 u}{\partial x \partial y \partial z}$	L3	CO3

1.h)	Find the stationary points of $f(x, y) = x^3 + y^3 - 3axy$ .	L2	CO5
1.i)	Compute $\int_0^a \int_0^b \int_0^c x^2 y^2 z^2 dx dy dz$ .	L3	CO3
1.j)	State the formula for area in polar coordinates.	L2	CO3

### PART – B

			BL	CO	Max. Marks
<b>UNIT-I</b>					
2	a)	By reducing into Normal form find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$	L3	CO2	5 M
	b)	Solve $3x + 2y + z = 0, x + 4y + z = 0, 2x + y + 4z = 0$	L3	CO4	5 M
<b>OR</b>					
3	a)	Find rank of $A = \begin{bmatrix} 2 & -1 & 3 & 1 \\ 1 & 4 & -2 & 1 \\ 5 & 2 & 4 & 3 \end{bmatrix}$ by reducing into Echelon form.	L3	CO2	5 M
	b)	Solve $x + y + z = 8$ $2x + 3y + 2z = 19$ $4x + 2y + 3z = 23$ by Gauss elimination method.	L3	CO4	5 M
<b>UNIT-II</b>					
4	a)	Calculate Eigen values and Eigen vectors of $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$	L3	CO2	5 M

	b)	Verify Cayley-Hamilton theorem for $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and hence find $A^{-1}$ and $A^4$ .	L3	CO2	5 M
<b>OR</b>					
5		Reduce the quadratic form $3x^2 + 3y^2 + 3z^2 + 2xy + 2xz - 2yz$ in to canonical form by orthogonal transformation method and find its rank, index, signature and nature.	L3	CO4	10 M
<b>UNIT-III</b>					
6	a)	Verify Rolle's theorem for the function $f(x) = x^2 - 6x + 8$ in the interval $[2, 4]$ .	L3	CO5	5 M
	b)	Expand $\sin x$ by Maclaurin's series up to the term containing $x^5$	L3	CO3	5 M
<b>OR</b>					
7	a)	Verify Lagrange's mean value theorem for the function $f(x) = x^3 - x^2 - 5x + 3$ in $[0, 4]$ .	L3	CO5	5 M
	b)	Verify Cauchy's mean value theorem for the function $f(x) = \sin x, g(x) = \cos x$ in the interval $\left[0, \frac{\pi}{2}\right]$ .	L3	CO5	5 M
<b>UNIT-IV</b>					
8	a)	If $z = \log(e^x + e^y)$ then show that $\left(\frac{\partial^2 z}{\partial x^2}\right)\left(\frac{\partial^2 z}{\partial y^2}\right) = \left(\frac{\partial^2 z}{\partial x \partial y}\right)^2$	L2	CO3	5 M
	b)	A rectangular box open at the top is to have volume of 32 cubic feet. Find the dimensions of the box required in least material for its construction.	L4	CO5	5 M

<b>OR</b>					
9	a)	If $z = f(x + ay) + g(x - ay)$ then show that $\frac{\partial^2 z}{\partial y^2} = a^2 \frac{\partial^2 z}{\partial x^2}$	L3	CO3	5 M
	b)	Show that the functions $u = xy + yz + zx$ , $v = x^2 + y^2 + z^2$ , $w = x + y + z$ are functionally dependent. Find the relation between them.	L3	CO3	5 M

### UNIT-V

10	a)	Evaluate $\iint_R r \sin \theta dr d\theta$ where $R$ is the cardioid $r = a(1 - \cos \theta)$ above the initial line.	L3	CO5	5 M
	b)	Evaluate $\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z} dz dy dx$	L3	CO5	5 M

### OR

11	a)	Evaluate $\iint_A xy dx dy$ , where $A$ is the domain bounded by x-axis, ordinate $x=2a$ and the curve $x^2 = 4ay$ .	L4	CO5	5 M
	b)	Calculate the area common to the circles $r = a \sin \theta$ and $r = a \cos \theta$ by double integration.	L3	CO5	5 M